# Sum rule analysis of vector and axial-vector spectral functions with excited states in vacuum 

P. M. Hohler and R. Rapp

At low temperatures and chemical potentials, the QCD vacuum breaks chiral symmetry with the formation of the quark (or chiral) condensate. This symmetry is believed to be restored at high temperatures, but a definitive experimental signature of this restoration has not been observed. A promising way to identify chiral symmetry restoration is through the medium modifications of the spectral functions of chiral partners, such as $\rho$ and $a_{1}$ mesons Much has been learned about the in-medium properties of the vector channel from dilepton data [1,2], but no experimental measurement of the inmedium axial-vector spectral function has been performed to date. To assist the experimental effort, theoretical tools can be used to relate the properties of the two channels so that chiral symmetry restoration can be inferred from existing data. One such tool are sum rules. Weinberg-type sum rules [3-5] relate the differences between the vector and axial-vector channels and to chiral order parameters, while QCD sum rules [6] are specific for each channel. Using the combination of these two sets of sum rules, one can hope to constrain and connect the in-medium spectral functions in the two channels. In order to identify the pertinent medium modifications, a firm understanding of the vacuum spectral functions is needed first.

In the present work [7], we simultaneously analyze vector and axial-vector spectral functions in vacuum using an extended model which combines a microscopic $\rho$ spectral function [8] with BreitWigner ansaetze for the $a_{1}$ and the first excited states. Our model is quantitatively constrained by both experimental $\tau$-decay data [9] and Weinberg sum rules. By using this combination of criteria, the model may be considered a non-trivial fit of the data, as displayed in Fig. 1. Novel features of our analysis include the study of excited states and the postulate that the high-energy continuum contribution is identical in vector and axial-vector channels, as should be the case in the perturbative regime. In particular, the use of the Weinberg sum rules leads us to infer the presence of an excited axial-vector state, $a_{1}{ }^{\prime}$, about which rather little is known to date. Quantitatively, the inclusion of the $a_{1}{ }^{\prime}$ improves the


FIG. 1. Spectral functions in the vector (left) and axial-vector channel (right) compared to experimental data for hadronic $\tau$-decays by the ALEPH collaboration [9]. The different curves highlight the contributions to the total spectral function (solid curve) from the ground-state resonance (dotted curve), the excited resonance (dashed curve), and the high-energy continuum (dot-dashed curve).
agreement of the four Weinberg-type sum rules from deviations of ( $3.6 \%, 43 \%, 7000 \%,-1400 \%$ ) down to $(-1.28 \%, \sim 0 \%, \sim 0 \%,-96 \%)$, where the positive (negative) sign indicates an excess in the vector (axialvector) channel. As an additional check, we utilize the constructed spectral functions within their respective QCD sum rules to check the lesser established gluon and four-quark condensates. Requiring an agreement of the QCD sum rules within $1 \%$ or less, we find the gluon condensate to be $(0.022 \pm 0.002)$ $\mathrm{GeV}^{4}$, while the four-quark condensate factorization parameter turns out to be (2.1+0.3-0.2). We believe that these spectral functions provide a useful basis for future studies of medium modifications in order to shed light on the long-standing problem of testing chiral symmetry restoration with dilepton data.
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